

QUALIFYING EXAM IN GEOMETRY AND TOPOLOGY

$$X_2 \cong \mathbb{R}^2 \cup \{0\} \cong \mathbb{R}^n \cup \{0\}$$

$$H_p(X_k, \mathbb{Z}_k) \cong H_p(X_k, \mathbb{S}^1) \cong \mathbb{Z}_k \oplus \mathbb{Z}_k \oplus \dots \oplus \mathbb{Z}_k \quad \text{for } p \geq 1$$

$$c_1 \in H^2(X) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \dots \oplus \mathbb{Z} \quad \text{for } f: M \rightarrow N$$

$$N \cong \mathbb{R}^n \cup \{0\}$$

$$n = \text{rank of } X \cong \mathbb{R}^n \cup \{0\} \cong \mathbb{R}^n \cup \{0\}$$

$$M \cong N \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \dots \oplus \mathbb{Z}$$

$$f: M \rightarrow N$$

$$\text{rank}_1 M \geq \text{rank}_1 N$$

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$$c_1 \in H^2(X) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \dots \oplus \mathbb{Z} \cong \mathbb{R}^3$$