

$\frac{1}{2} q^a \cdot 2.]$

Exercise 2. Let R be an integral domain that is integrally closed in its field of fraction F .

- (1) Show that an algebraic element is integral over R if and only if its minimal polynomial over F is a monic polynomial in $R[x]$.
- (2) Show that for any monic $f(x) \in R[x]$, for any decomposition $f(x) = f_1(x)f_2(x)$ into monic polynomials in $F[x]$, the factors f_1, f_2 have coefficients in R .

Exercise 3. Let k be an algebraically closed field. Consider the affine variety $V = k^2$ (with coordinates x, y), and the affine variety $W = k^2$ (with coordinates s, t). Suppose $\phi : V \rightarrow W$ is a morphism, and denote by $R = k[x, y]$ the image of the induced ring homomorphism $\phi^* : k[s, t] \rightarrow k[x, y]$. For each of the following statements, give a proof or a counterexample.

- (1) If ϕ has Zariski dense image, then ϕ^* is surjective.
- (2) If $k[x, y] = R$ is any integral extension of $k[s, t]$, then ϕ^* is surjective.

Exercise 7. Suppose k be a field and $R = k[x; y; z]$ a polynomial ring. Compute

$$\text{Ext}_R^i R/(xz); R/(xy; xz)$$

for all $i \geq 0$.

Exercise 8. Suppose p is a prime of the form $4k + 3$. Find the conjugacy class of every element of order 4 in $\text{GL}_2(\mathbb{F}_p)$.